

Exponential and logarithm Functions

1 a) $e^{2x+1} \ln(3x)$

Differentiating using chain rule:

$$= \frac{d}{dx} (e^{2x+1}) \ln(3x) + e^{2x+1} \frac{d}{dx} (\ln 3x)$$

$$= 2e^{2x+1} \ln(3x) + e^{2x+1} \left(\frac{1}{3x}\right)$$

$$= 2e^{2x+1} (\ln 3x) + \frac{1}{x} e^{2x+1}$$

b) 3^{4x}
 $\frac{d}{dx} (3^{4x})$

$$= \ln(3) 3^{4x} \frac{d}{dx} (4x)$$

$$= 4 \ln(3)$$

$$= \ln(3) \cdot 4 \cdot \frac{d}{dx} (x) 3^{4x}$$

$$= 4 \ln(3) 3^{4x}$$

$$= 4 \ln(3) 3^{4x}$$

2 ① $\frac{x-1}{x^3}$

$$= \frac{\frac{d}{dx} (x-1) x^3 - (x-1) \frac{d}{dx} (x^3)}{(x^3)^2}$$

$$= \frac{(1+0)x^3 - 3(x-1)x^2}{x^6}$$

$$= \frac{x^3 - 3(x-1)x^2}{x^6}$$

$$2. \quad 4e^{4x} - 4e^{2x} = 0$$

$$e^{4x} = 4e^{2x}$$

$$\ln(e^{4x}) = \ln 4 + \ln(e^{2x})$$

$$4x = \ln 4 + 2x$$

$$2x = \ln 4$$

$$x = \ln 4 / 2$$

$$x = 2 \quad x' = 0.6931$$

$$b) \quad \ln x + \ln(x+1) = \ln 2$$

$$\ln x + \ln(x+1) = \ln\left(\frac{x}{x+1}\right) = \ln 2$$

$$\exp\left[\ln\left(\frac{x}{x+1}\right)\right] = \exp[\ln 2]$$

$$\frac{x}{x+1} = 2$$

$$x = 2x + 2$$

$$-x = 2$$

$$x = -2$$

$$4. \quad y = e^x + e^{-x}$$

$$f'(x) = 0$$

finding the derivative and equating to zero

$$f'(x) = e^x + e^{-x} = 0$$

$$e^x - e^{-x} = 0$$

$$\ln(e^x) - \ln(e^{-x}) = 0$$

$$x + x = 0$$

$$2x = 0$$

$$y = e^0 + e^0 = 1$$

turning point (0, 1)

$x < 0$ is relative minimum.

$$5 \quad y = Ate^{-bt}$$

$$a) \quad \ln y = \ln A + \ln t - bt = 0$$

$$f'(t) = 0 + \frac{1}{t} - b = 0$$

$$\frac{1}{t} - b = 0$$

$$\frac{1}{t} = b$$

$$\text{Turning point; } t = \frac{1}{b}$$

$$b) \quad f'(t) = \frac{1}{t} - b$$

$$f''(t) = -\frac{1}{t^2} = 0$$

+

$$-\ln 2t - \ln b = 0$$

$$-\ln 2t = \ln b$$

$$t = \frac{2}{b}$$

c). Turning point

$$t = \frac{1}{b}$$

$$y = Ate^{-bt}$$

$$= A\left(\frac{1}{b}\right)e^{-bt}$$

point of inflection

$$t = \frac{2}{b}$$

$$y = A\left(\frac{2}{b}\right)e^{-bt}$$

$$b) P(t) = \frac{10000}{1 + 1000e^{-0.2t}}$$

a) Initial population, $t = 0$.

$$\begin{aligned} P(0) &= \frac{10000}{1 + 1000e^0} \\ &= 9.99 \\ &= 10. \end{aligned}$$

b) Carrying capacity, $t = \infty$

$$\begin{aligned} P(\infty) &= \frac{10000}{1 + 1000e^\infty} \\ &= 10000 \end{aligned}$$

$$c) 5000 = \frac{10000}{1 + 1000e^{0.2t}}$$

$$5000 + 5000000e^{0.2t} = 10000$$

$$e^{0.2t} = 0.001$$

$$-0.2t = \ln 0.001$$

$$t = 34.54 \text{ years}$$

$$y_t = b$$

$$d) 9000 = \frac{10000}{1 + 1000e^{-0.2t}}$$

$$9000 + 9000000e^{-0.2t} = 10000$$

$$e^{-0.2t} = 0.000111$$

$$-0.2t = \ln(0.000111)$$

$$t = 45.59 \text{ years}$$

$$= 46 \text{ years}$$